DETERMINATION OF THE SUPPORTING CAPACITY OF LAYERED COMPOSITE SHELLS OPERATING UNDER CYCLIC LOADING CONDITIONS

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In view of the considerable expansion of the field of use of composites in technology and an increase in their weight fraction within the total volume of material consumed in preparing articles it is very important to predict the strength characteristics of structures made of composites with short-term, prolonged, and cyclic loads. There is an extensive list of literature on this problem. The scientific bases of the calculation are given in [1, 2]. Experimental determination of the fatigue characteristics of composites which have a relatively simple structure were the subject in [3-6].

A procedure is suggested in the present work for finding the supporting capacity of thin layered composite shells with cyclic loading by proceeding from the corresponding fatigue characteristics, thickness, and orientations of individual layers in a composite. Layered composites formed by superimposing quasiuniform orthotropic layers are considered. Assumptions about the nature of the stress—strained state which are made by us do not limit the applicability of this procedure.

A layer with number j ($j = \overline{1, n}$, n is number of layers) is referred to a coordinate system $(Oxyz)_j$ whose axes coincide with the orthotropic axes of the j-th layer (z_j is orthogonal to its central surface). We introduce a global system $O\xi_1\xi_2 z$, the directions of whose axes ξ_1, ξ_2 are connected with the principal lines of curvature of the shell. The orientation of the j-th layer in the structure of a composite shell is determined by the value of angle φ_j, read from ξ_1 to x_j. The effect of stress tensor components σ_{xz}, σ_{yz}, σ_{zz} on the failure process is ignored. Maximum and minimum stresses in the cycle are derived by the well-known equation

$$\sigma^{\alpha\beta}_{\substack{\max\\ \min}} = \sigma^{\alpha\beta}_m \pm \sigma^{\alpha\beta}_a, \quad \alpha,\ \beta = x,\ y \vee 1,\ 2.$$

Here $\sigma_m{}^{\alpha\beta}$ are average stresses in the cycle; $\sigma_a{}^{\alpha\beta}$ are amplitude values of the variable part of stresses (in future the same index may occupy both the upper and lower position).

The condition for reaching the limiting state for the j-th layer with prescribed endurance N (N is number of cycles to failure) is written in the form

$$\begin{gathered}(a_1\sigma^2_{xx} + 2a_2\sigma_{xx}\sigma_{yy} + a_3\sigma^2_{yy} + 2a_4\sigma_{xx} + \\ + 2a_5\sigma_{yy} + a_6\sigma^2_{xy})^j_m + (b_1\sigma^2_{xx} + 2b_2\sigma_{xx}\sigma_{yy} + b_3\sigma^2_{yy} + b_4\sigma^2_{xy})^j_a = 1.\end{gathered} \tag{1}$$

In the space of values $\sigma_m{}^{\alpha\beta}$, $\sigma_a{}^{\alpha\beta}$ Eq. (1) corresponds to some surface (hypersurface) which is a generalized limiting amplitude diagram (Hay diagram). In recording (1) assumptions are considered about the nature of the stress—strain state. The requirement for invariance of (1) with respect to transformation of x_j into $-x_j$ and y_j into $-y_j$ leads to the requirement for equality to zero of terms linear with respect to σ_{xy}. With $\sigma_m{}^{\alpha\beta} \equiv 0$ (α, β = x, y) (1) should describe a surface in the space of $\sigma_a{}^{\alpha\beta}$ whose center of symmetry coincides with the origin. Therefore, there are no terms in it containing only $\sigma_a{}^{xx}$, $\sigma_a{}^{yy}$. The section of surface (1) with planes ($\sigma_m{}^{\alpha\beta}$, $\sigma_a{}^{\alpha\beta}$) (α, β = x, y) should be symmetrical with respect to the axis of $\sigma_m{}^{\alpha\beta}$ (for this reason it is only possible to depict the Hay diagram for positive σ_a in system $\sigma_m O \sigma_a$), and therefore there are terms in (1) with derivatives $\sigma_m{}^{\alpha\beta}\sigma_a{}^{\alpha\beta}$. Coefficients $a_1{}^j$, ..., $a_6{}^j$ in (1) are determined from the results of testing with short-term static

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loading, and coefficients $b_1{}^j(N)$, ..., $b_4{}^j(N)$ are determined with symmetrical cycles. In order to find $b_1{}^j$, ..., $b_4{}^j$ it is necessary to plot fatigue curves (Weller curves). In order to plot each individual fatigue curve it is necessary by retaining a constant direction for the 'vector' with components of $\sigma_a{}^{\alpha\beta}$, to change only its length and to calculate the corresponding endurance values. By analogy with the concepts of characteristic loading paths and characteristic strengths [1, p. 241] it is useful to introduce concepts of characteristics directions for the 'vector' with components of $\sigma_a{}^{\alpha\beta}$ and characteristic fatigue curves. Analytical approximation of experimental fatigue curves (or their individual sections) makes it possible to consider coefficients $b_1{}^j$, ..., $b_4{}^j$ as functions of endurance N.

Formally we reduce the problem of the supporting capacity of layered composite shells with cyclic loading to the problem of their strength with short-term static loading. In (1) the group of terms contained in the second bracket we denote as

$$\omega_a^j = \left(b_1\sigma_{xx}^2 + 2b_2\sigma_{xx}\sigma_{yy} + b_3\sigma_{yy}^2 + b_4\sigma_{xy}^2\right)_a^j.$$

Let the terms for stresses $\sigma_a{}^{\alpha\beta}$ enter into a collection of original conditions of the problem (permissible regions of their values are established using the inequality $0 \le \omega_a{}^j \le 1$). Taking this into account (1) assumes the form

$$\left(a_1\sigma_{xx}^2 + 2a_2\sigma_{xx}\sigma_{yy} + a_3\sigma_{yy}^2 + 2a_4\sigma_{xx} + 2a_5\sigma_{yy} + a_6\sigma_{xy}^2\right)_m^j = 1 - \omega_a^j. \tag{2}$$

It is suggested that after determining the number of cycles of force action the process of damage accumulation (scattered throughout the volume) reaches that level when the effective cross section (for the solid part of the material) becomes quite small, and the level of stresses in the material reaches a critical value which leads to intense deformation and subsequent failure. The condition of the material with which intense development of deformation commences (strain rates are high) is naturally taken as the post-limit condition (in time it immediately precedes endurance N). On the basis of this in order to solve the stated problem it is possible to use a procedure given in detail in [7]. The criterion for loss of supporting capacity (2) in the system

$$\begin{aligned}\Phi_j = \big(&A\sigma_{11}^2 + 2B\sigma_{11}\sigma_{22} + C\sigma_{22}^2 + 2D\sigma_{11} + \\ &+ 2E\sigma_{22} + L\sigma_{12}^2 + 2P\sigma_{11}\sigma_{12} + 2R\sigma_{22}\sigma_{12} + 2Q\sigma_{12}\big)_m^j + F_j = 0,\end{aligned} \tag{3}$$

where $F_j = \omega_a{}^j - 1$; A_j, ..., Q_j depend linearly on $a_1{}^j$, ..., $a_6{}^j$ and are functions of the orientation of the j-th layer in bundle φ_j. Equation (3) is an equation of the limiting surface in the space of values of $\sigma_m{}^{\alpha\beta}$ (the free term depends on prescribed values of $\sigma_a{}^{\alpha\beta}$). We assume that the limiting material condition is stable, i.e., the Drucker postulation is valid [8]. Whence it follows that the limiting surface should be convex. Stresses $\sigma_m{}^{\alpha\beta}$ enter as generalized forces [8]. The velocity vector for the corresponding generalized displacements (with components $\dot{\varepsilon}_{\alpha\beta}$) according to the Drucker postulation should be directed along the external normal to surface (3):

$$\dot{\varepsilon}_{\alpha\beta}^j = \left(\dot{\lambda}\,\partial\Phi/\partial\sigma_m^{\alpha\beta}\right)_j, \quad j = \overline{1,\ n}; \quad \alpha,\ \beta = 1,\ 2. \tag{4}$$

$\dot{\varepsilon}_{\alpha\beta}$ should not be identified with traditional strain rates for a solid. Here they enter into the role of kinematic parameters representing generalized displacement rates which satisfy the condition $d\sigma d\varepsilon \ge 0$ (hypothesis). We write briefly the further course of solving the problem (see [7]). By using (3) and (4) we determine $\dot{\varepsilon}_{\alpha\beta}{}^j$. We solve the set of equations obtained with respect to $(\sigma_m{}^{\alpha\beta})_j$. By substituting $(\sigma_m{}^{\alpha})_j$ in (3) we express $\dot{\lambda}_j \ge 0$ in terms of $\dot{\varepsilon}_{\alpha\beta}{}^j$. We assume that in expanding kinematic characteristics $\dot{\varepsilon}_{\alpha\beta}{}^j$ into a series with respect to z it is sufficient to retain the linear part (hypothesis):

$$\dot{\varepsilon}_{\alpha\beta}^j = \dot{e}_{\alpha\beta} - z\dot{\varkappa}_{\alpha\beta}, \quad \alpha,\ \beta = 1,\ 2$$

(axis z is orthogonal to the reduction surface of the shell S_0, where z = 0). We calculate the static terms of linear forces and moments reduced to S_0:

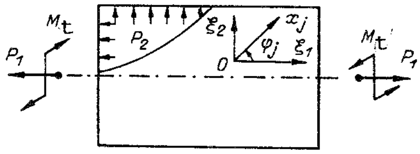

Fig. 1

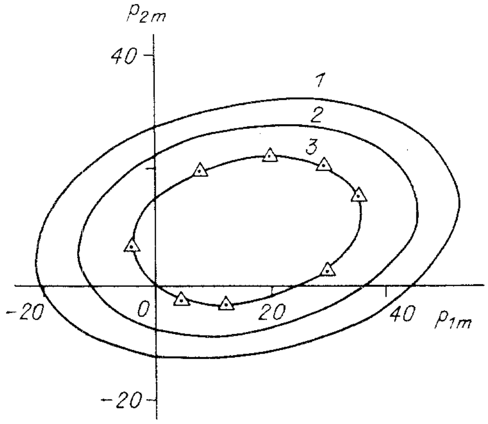

Fig. 2

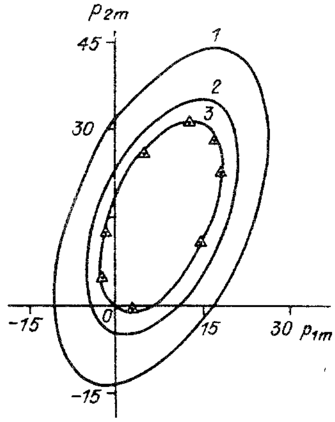

Fig. 3

$$T_m^{\alpha\beta} = \sum_{j=1}^{n} \int_{z_{1j}}^{z_{2j}} \sigma_m^{\alpha\beta}\, dz, \quad M_m^{\alpha\beta} = \sum_{j=1}^{n} \int_{z_{1j}}^{z_{2j}} \sigma_m^{\alpha\beta} z\, dz,$$

where $z_{1j} < z_{2j}$ are the coordinates of points which lie at the restraining surfaces of the j-th layer; α, β = 1, 2. As a result of this

$$\begin{aligned} T_m^i &= \sum_{j=1}^{n} \left[0.5 \sum_{k=1}^{3} \delta_{ik}^j \left(I_{1j}\dot{e}_k - I_{2j}\dot{\varkappa}_k \right) - \Delta_{ij} h_j \right] \Big/ \Delta_j; \\ M_m^i &= \sum_{j=1}^{n} \left[0.5 \sum_{k=1}^{3} \delta_{ik}^j \left(I_{2j}\dot{e}_k - I_{3j}\dot{\varkappa}_k \right) - \Delta_{ij} h_j z_j \right] \Big/ \Delta_j. \end{aligned} \tag{5}$$

Here $i = \overline{1, 3}$; $T_\alpha \equiv T_{\alpha\alpha}$; $M_\alpha \equiv M_{\alpha\alpha}$; $\dot{e}_\alpha \equiv \dot{e}_{\alpha\alpha}$; $\dot{\varkappa}_\alpha \equiv \dot{\varkappa}_{\alpha\alpha}$ (α = 1, 2); $T_3 \equiv T_{12}$; $M_3 \equiv M_{12}$; $\dot{e}_3 \equiv \dot{e}_{12}$; $\dot{\varkappa}_3 \equiv \dot{\varkappa}_{12}$; $h_j = (z_{2j} - z_{1j})/2$; $z_j = (z_{2j} + z_{1j})/2$; Δ_j, Δ_{ij}, $\delta_{ik}{}^j$ are determinants whose elements are coefficients from (3) (see [7]). Integrals I_{ij} are calculated according to the expression

$$I_{ij} = \int_{z_{1j}}^{z_{2j}} (z^{i-1}/\dot{\lambda}_j)\, dz, \quad i = \overline{1, 3}.$$

Equations (5) are parametric equations of the limiting surface for layered composite shells in a space of static terms of forces and moments. Values of $T_m{}^{\alpha\beta}$, $M_m{}^{\alpha\beta}$ in (5) operating together with cyclic forces and moments (their amplitude values are determined by terms for stresses $\sigma_a{}^{\alpha\beta}$) on reaching the number of loading cycles N bring the composite shell to fatigue failure. Hypersurface (5) in the space of values $T_m{}^{\alpha\beta}$, $M_m{}^{\alpha\beta}$ is a section of the more general limiting surface (surface of static strength and limited endurance) with a hypersurface passing through the end of the 'vector' whose components are determined by amplitude values of $\sigma_a{}^{\alpha\beta}$ of the variable part of stresses orthogonal to this 'vector.'

With the aim of reducing the amount of experimental work required for calculating the coefficients of Eq. (1) it is possible to use conclusions given in [4] where it was shown that stable correlation between σ_{-1} and $\sigma_{min}{}^*$ (σ_{-1} is fatigue strength with endurance N corresponding to symmetrical cycles, $\sigma_{min}{}^*$ is the least of the limits of proportionality with static tension and compression along the same direction) does not depend on the direction of cutting specimens for a given reinforced plate. This may be used for considerable shortening of the number of fatigue tests for orientated composite materials.

As an example of using relationships (5) we give the solution of the problem of the limiting condition of a cylindrical shell with stiffened ends. The shell is loaded by axial force P_1, internal pressure P_2, and torsional moment M_t (positive loading directions are shown in Fig. 1). We introduce dimensionless parameters $\sigma_{\alpha\beta}^* = \sigma_{\alpha\beta}/\sigma_0$, $t_{\alpha\beta} = T_{\alpha\beta}/\sigma_0 H$, $m_t = M_t/2\sigma_0 HA$, $p_1 = P_1/\pi dH\sigma_0$, $p_2 = P_2(d - H)^2/2dH\sigma_0$. Here σ_0 is a value having the dimension of stresses; H is shell thickness; d is its diameter; A is an area bounded by the contour of a transverse section of the shell. Statics equations give the following relationships between parameters of external loads and internal forces:

$$p_1 = t_{11} - 0.5t_{22},\ p_2 = t_{22},\ m_t = t_{12}. \tag{6}$$

The criterion for loss of supporting capacity for the j-th layer with cyclic loading has the form (for convenience the * for dimensionless stresses is omitted)

$$(1.50\sigma_{xx}^2 - 2.45\sigma_{xx}\sigma_{yy} + 4.78\sigma_{yy}^2 - 38.2\sigma_{xx} - 40.7\sigma_{yy} + \\ + 26.1\sigma_{xy}^2)_m^j = 1399.3 - \omega_a^j, \tag{7}$$

where

$$\omega_a^j = (28.2\sigma_{xx}^2 - 39.9\sigma_{xx}\sigma_{yy} + 81.6\sigma_{yy}^2 + 1399.3\sigma_{xy}^2)_a^j. \tag{8}$$

In writing (7) and (8) experimental results provided in [5] for a test base of $N = 10^6$ cycles (for standard fiber glass laminate VFT-S) are used. Stresses are referred to $\sigma_0 = \sigma_{-1}{}^{xy} = 8.45$ MPa.

Given in Figs. 2 and 3 are different limiting curves plotted in the plane of parameters (p_{1m}, p_{2m}) using relationships (5)-(8) (static terms are denoted by index m, and amplitude values of the variable part of the load are denoted by index a). Results given in Fig. 2 were obtained for cases when the orientation angle φ_j for all of the layers equals zero, and in Fig. 3 for shells in which layers with a winding angle +45° alternate with layers with an angle of −45°. Everywhere the number of layers equals eleven. Curves 1 in Figs. 2 and 3 correspond to the case when the cycle amplitude equals zero (strength curves with short-term static loading). Curves 2 and 3 in Fig. 2 relate to amplitude values of the variable part of load parameters $p_{1a} = 5$, $p_{2a} = m_{ta} = 0$ and $p_{1a} = 7.035$, $p_{2a} = m_{ta} = 0$, respectively. Another curve is placed very close to curve 3 when parameters of the amplitude components of loads are as follows: $p_{1a} = p_{2a} = 0$, $m_{ta} = 0.99$ (some of its points are placed within triangles). Similarly in Fig. 3 curves 2 and 3 with $p_{1a} = 1.5$, $p_{2a} = m_{ta} = 3$ and $p_{1a} = p_{2a} = 0$, $m_{ta} = 3$. Another curve is placed very close to curve 3 in Fig. 3 with the parameters $p_{1a} = -1.435$, $p_{2a} = m_{ta} = 0.95$ (some of its points are placed within triangles).

On the basis of the results provided above it is possible to conclude that the method developed for determining the supporting capacity of layered composite shells with cyclic loading makes it possible to obtain a qualitatively correct picture of this phenomenon. The procedure gives reference points in determining the parameters of reinforced materials in order to find the fatigue strength of structures prepared from these materials. In fact, in order to check the quantitative conformity of the results obtained it is necessary to carry out new experimental studies.

The method suggested also helps in carrying out the reverse course: from tests on a layered composite, i.e., a specimen with a prescribed structure, to draw conclusions about the properties of a monolayer and then to apply these results by the method suggested to a composite with another layer configuration.

LITERATURE CITED

1. A. K. Malmeister, V. P. Tamuzh, and G. A. Teters, Strength of Polymer and Composite Materials, 3rd ed. [in Russian], Zinatne, Riga (1980).
2. Yu. V. Nemerovskii and B. S. Reznikov, Strength of Structural Elements Made of Composite Materials [in Russian], Nauka, Novosibirsk (1986).
3. E. K. Ashkenazi and A. A. Pozdnyakov, "Fatigue testing of glass fiber laminates," Zavod. Lab., 27, No. 10 (1961).

4. P. P. Oldyrev, "Correlation between the static and fatigue strengths of reinforced plastics," Mekh. Polim., No. 3 (1973).
5. P. P. Oldyrev, "Evaluation of fatigue strength anisotropy for composite materials," Mekh. Kompozit. Materialov, No. 1 (1982).
6. I. P. Petrenko, "Fatigue strength diagrams for fiber glass laminate VFT-S with repeated-alternating axial loading," Dokl. Akad. Nauk UkrSSR, No. 8 (1965).
7. I. G. Teregulov, É. S. Sibgatullin, and O. A. Markin, "Limiting state of multilayer composite shells," Mekh. Kompozit. Materialov, No. 4 (1988).
8. M. I. Erkhov, Theory of Ideally Plastic Bodies and Structures [in Russian], Nauka, Moscow (1978).

APPROXIMATE ANALYSIS OF THE THERMAL OPERATING REGIME OF A CHAMBER FOR TREATING MATERIALS WITH THERMAL ENERGY*

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A thermal operating regime is studied for the walls of a chamber for deburring materials. Often these chambers are made in the form of thin-walled cylindrical vessels whose walls have thermal protection (a thin internal layer of material with high thermal conductivity). The chamber wall is modeled by an infinite two-layer plate, the external surface of the plate is maintained at a prescribed temperature, and at the internal surface the heat flow is prescribed [1] characterizing the heat transfer of gas mixture detonation products into the chamber wall. It is assumed that at the interface of the layers a condition of ideal thermal contact is fulfilled. The chamber operating regime for deburring materials is defined by the periodicity of treatment cycles which as a rule are 15-20 sec.

The temperature field for the chamber wall is constructed both for the first cycle and with several of the first treatment cycles. The construction with the first cycle is carried out by two standard methods. It is revealed that introducing thermal protection makes it possible to reduce by several factors the temperature of the internal surface and to increase by about an order of magnitude the time for reaching the maximum temperature of the internal surface compared with a single-layer chamber.

The temperature field in a time interval corresponding to five material treatment cycles is obtained by the method of numerical modeling (Fig. 1). The broken line relates to a single-layer chamber wall, and the solid line relates to a two-layer chamber. It can be seen

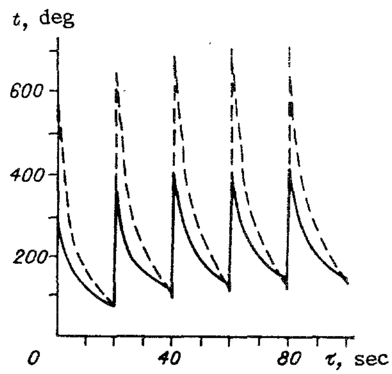

Fig. 1

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